

Unfortunately, this constraint is not a direct measure of the fuel consumed and leads to a thrust program quite different from the optimum program with a fixed amount of fuel

The constraint used is the sum of the squares of the velocity impulses, whereas the fuel consumption depends upon the sum of the absolute magnitudes of the velocity impulses. For a fixed value of the constraint, the sum of the absolute magnitudes must lie somewhere between the square root of the sum of the squares of the impulses and the square root of the number of impulses multiplied by the square root of the sum of the squares of the impulses. The particular value of the fuel consumption for a given value of the constraint depends upon the distribution of the impulses.

An analytic solution has been presented for maximizing the changes in the three orbital parameters considered in Ref 1 with a fixed amount of fuel²⁻⁴. The solution of this more practical problem leads to impulses applied only at the line of nodes. The solution obtained numerically with this particular constraint on fuel consumption leads to impulses distributed all around the orbit and is quite different from the analytic solution with a given amount of fuel.

The value of this particular constraint on fuel consumption appears to be rather limited.

References

- ¹ Burkhart, J. A. and Smith, F. T., "Application of dynamic programming to optimizing the orbital control process of a 24-hour communications satellite," *AIAA J* 1, 2551-2557 (1963).
- ² Edelbaum, T. N., "Propulsion requirements for controllable satellites," *ARS J* 31, 1079-1089 (1961).
- ³ Fiul, A. and Braham, H., "Optimization of vehicles and trajectories for the twenty-four hour equatorial satellite mission," *ARS Preprint* 1120 60 (1960).
- ⁴ Hoelker, R. F. and Silber, R., "Analysis of injection schemes for obtaining a twenty four hour orbit," *Aerospace Eng* 20, 28-29, 76-82 (January 1961).

Comments on "Radiation Slip"

A. G. HAMMITT*

TRW Space Technology Laboratories,
Redondo Beach, Calif

THE purpose of this note is to suggest an improvement of the method that appears in Ref 1 and to point out some of the limitations of this approach.

In the radiation problem, the fourth power of the temperature plays the analogous role to the first power of the velocity in the normal viscous diffusion of momentum or the first power of the temperature in the thermal conduction of heat. This analogy is obvious if Eqs (4) and (5) of Ref 1 are combined to give

$$-q = \frac{4}{3}\sigma l(dT^4/dy) = \frac{4}{3}\sigma(dT^4/d\tau)$$

using the notation of Ref 1. The analogous slip condition becomes

$$T^4 - T_w^4 = Kl(dT^4/dy) = K(dT^4/d\tau)$$

This condition gives Eq (9) of Ref 1 for all values of slip, whereas Eq (8) of Ref 1 does not, except in the limit of small slip. Since Eq (8) is not used again, but all of the results are based on Eq (9), this change does not affect the results presented except that it may make the conclusion that Eq (11) gives good results even for large slip a little more reasonable.

However, difficulty is encountered if Eq (8) is used to calculate the temperature jump at the wall, but this difficulty is eliminated by the suggested improvement. A comparison of the temperature profiles given by the slip method using the improved boundary conditions and the numerical calculations of Ref 2 is shown in Fig 1. The agreement of the suggested method with the numerical calculations is quite good, whereas using Eq (8) to calculate the temperature jump at the wall will cause large errors except for very small jumps.

It also is useful to point out some of the limitations of this method. In the example chosen, the method was applied to a Couette problem, where good agreement with exact results was obtained similar to the situation for the conduction of heat through a low-density gas. Difficulty has been encountered in applying these methods directly to more complicated problems, such as the Rayleigh problem, for low-density gas flow, and somewhat similar difficulties occur for radiation. The difficulty in the case of radiation is connected with the equating of the photon and gas temperature. In the absence of conduction, the gas temperature is unimportant in the Couette problem, since the gas is not absorbing or giving off heat. This condition does not

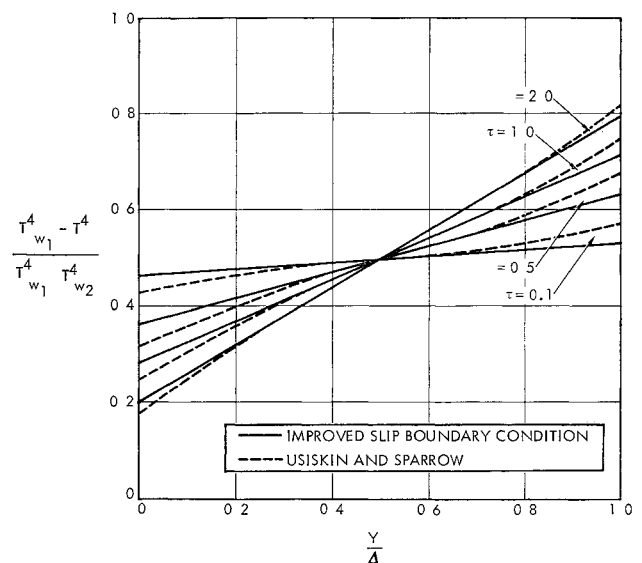


Fig 1

hold for the Rayleigh problem. To demonstrate the difficulty, if the photon temperature is set equal to the gas temperature in the Rayleigh problem, and $K = \frac{2}{3}$ (as in Ref 1), the initial heat transfer is twice the blackbody value.

Another limitation is imposed by the separation of the conduction and radiation heat-flow process to derive Eq (12). This separation can only be rigorously defended for the limits of a transparent or opaque gas. For the transparent gas, there is no connection between the two modes of heat transfer, and the heat flux by each mode is constant across the gap. For the opaque gas, when integrated with respect to temperature, the integral may be formally separated into the two terms, but the actual heat fluxes carried by radiation and conduction are not individually constant across the gap; only their sum is constant. Therefore, Eq (12) must be considered a convenient expression giving the correct limits, but the justification for using it at intermediate values of τ cannot be defended by the derivation.

References

- ¹ Probstein, R. F., "Radiation slip," *AIAA* 1, 1202-1204 (1963).
- ² Usiskin, C. M. and Sparrow, E. M., "Thermal radiation between parallel plates separated by an absorbing emitting non-isothermal gas," *Intern J Heat Mass Transfer* 1, 28-36 (1960).

Received January 8, 1964

* Manager, Aerosciences Laboratory Research Staff Member AIAA